MAXIMUM PRINCIPLE FOR NON-LINEAR
DEGENERATE INEQUALITIES OF PARABOLIC TYPE

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In recent years the maximum principle was extended to degenerate
elliptic parabolic equations, and has been studied by several authors,
for example in [2], [3], [4], [5], [6], [8], [9], [10]. In this paper we consider a differential inequality

(1) \[ \alpha(t,x)u_t - f(t,x,u(t,x),Du(t,x),D^2u(t,x)) \leq \]
\[ \alpha(t,x)v_t - f(t,x,v(t,x),Dv(t,x),D^2u(t,x)) \]

in \( Q = (0,T) \times \Omega \), where \( \Omega \) is an open and bounded set in \( \mathbb{R}^n \), and \( \alpha(t,x) \geq 0 \) in \( Q \). \( Du \) denotes the gradient of \( u \) with respect to \( x \), \( D^2u \) is the Hessian matrix of the second order derivatives (also with respect to the variable \( x \)). \( f(t,x,u,p,r) \) is assumed to be defined for \( (t,x) \in \phi \), \( u \in \mathbb{R} \), \( p \in \mathbb{R}^n \) and \( r \in \mathbb{R}^n \).

The main assumptions are that (i) \( f \) is weakly parabolic in sense of Besala (see [1]) (ii) \( f \) is decreasing with respect to \( u \), (iii) \( f \) is Lipschitz with respect to \( p \) and \( r \) and (iv) there exists a positive constant \( h \) and non-negative function \( \gamma \) such that

(2) \[ \alpha(t,x) + \gamma(t,x) \geq h \]

for all \( (t,x) \in Q \). If \( u \) and \( w \) are regular, (for definition see [7]) satisfy (1) and \( u - v \) has a non-negative maximum on \( \overline{Q} \) then this maximum is attained at some point of the parabolic boundary of \( Q \). Simple examples
show that one cannot expect a strong maximum principle with the assumption (2). Our weak maximum principle leads immediately to the uniqueness of the Dirichlet problem for the equation

$$\alpha(t,x)u_t = f(t,x,u,Du,D^2u)$$

to the uniqueness of the Cauchy problem for equation (3) in the class of functions which decay at infinity. With various additional assumptions concerning $f$ we obtain uniqueness of the Cauchy problem in classes of functions which grow at infinity not faster than

$$a) \ M \exp k |x|^2, \quad b) \ M \exp (k \sum_{i=1}^{n} |x_i|).$$

REFERENCES


